



22137404



**MATHEMATICAL STUDIES  
STANDARD LEVEL  
PAPER 2**

Friday 10 May 2013 (morning)

1 hour 30 minutes

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**INSTRUCTIONS TO CANDIDATES**

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **Mathematical Studies SL information booklet** is required for this paper.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is [90 marks].

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 11]

An agricultural cooperative uses three brands of fertilizer, A, B and C, on 120 different crops. The crop yields are classified as High, Medium or Low. The data collected are organized in the table below.

	Fertilizer			Total
	A	B	C	
High Yield	10	8	12	30
Medium Yield	24	14	12	50
Low Yield	16	8	16	40
Total	50	30	40	120

The agricultural cooperative decides to conduct a chi-squared test at the 1 % significance level using the data.

- (a) State the null hypothesis,  $H_0$ , for the test. [2 marks]
- (b) Write down the number of degrees of freedom. [1 mark]
- (c) Write down the critical value for the test. [1 mark]
- (d) Show that the expected number of Medium Yield crops using Fertilizer C is 17, correct to the nearest integer. [2 marks]
- (e) Use your graphic display calculator to find for the data
  - (i) the  $\chi^2$  calculated value,  $\chi^2_{calc}$ ;
  - (ii) the  $p$ -value. [3 marks]
- (f) State the conclusion of the test. Give a reason for your decision. [2 marks]

2. [Maximum mark: 16]

100 students at IB College were asked whether they study Music (*M*), Chemistry (*C*), or Economics (*E*) with the following results.

- 10 study all three
- 15 study Music and Chemistry
- 17 study Music and Economics
- 12 study Chemistry and Economics
- 11 study Music **only**
- 6 study Chemistry **only**

- (a) Draw a Venn diagram to represent the information above. [4 marks]
- (b) Write down the number of students who study Music but not Economics. [1 mark]

There are 22 Economics students **in total**.

- (c) (i) Calculate the number of students who study Economics only.
- (ii) Find the number of students who study none of these three subjects. [4 marks]

A student is chosen at random from the 100 that were asked above.

- (d) Find the probability that this student
  - (i) studies Economics;
  - (ii) studies Music and Chemistry but not Economics;
  - (iii) does not study either Music or Economics;
  - (iv) does not study Music given that the student does not study Economics. [7 marks]

3. [Maximum mark: 23]

George leaves a cup of hot coffee to cool and measures its temperature every minute. His results are shown in the table below.

Time, $t$ (minutes)	0	1	2	3	4	5	6
Temperature, $y$ ( $^{\circ}\text{C}$ )	94	54	34	24	$k$	16.5	15.25

- (a) Write down the decrease in the temperature of the coffee
- (i) during the first minute (between  $t = 0$  and  $t = 1$ );
  - (ii) during the second minute;
  - (iii) during the third minute. [3 marks]
- (b) Assuming the pattern in the answers to part (a) continues, show that  $k = 19$ . [2 marks]
- (c) Use the **seven** results in the table to draw a graph that shows how the temperature of the coffee changes during the first six minutes.  
Use a scale of 2 cm to represent 1 minute on the horizontal axis and 1 cm to represent  $10^{\circ}\text{C}$  on the vertical axis. [4 marks]

The function that models the change in temperature of the coffee is  $y = p(2^{-t}) + q$ .

- (d) (i) Use the values  $t = 0$  and  $y = 94$  to form an equation in  $p$  and  $q$ .
- (ii) Use the values  $t = 1$  and  $y = 54$  to form a second equation in  $p$  and  $q$ . [2 marks]
- (e) Solve the equations found in part (d) to find the value of  $p$  and the value of  $q$ . [2 marks]

The graph of this function has a horizontal asymptote.

- (f) Write down the equation of this asymptote. [2 marks]

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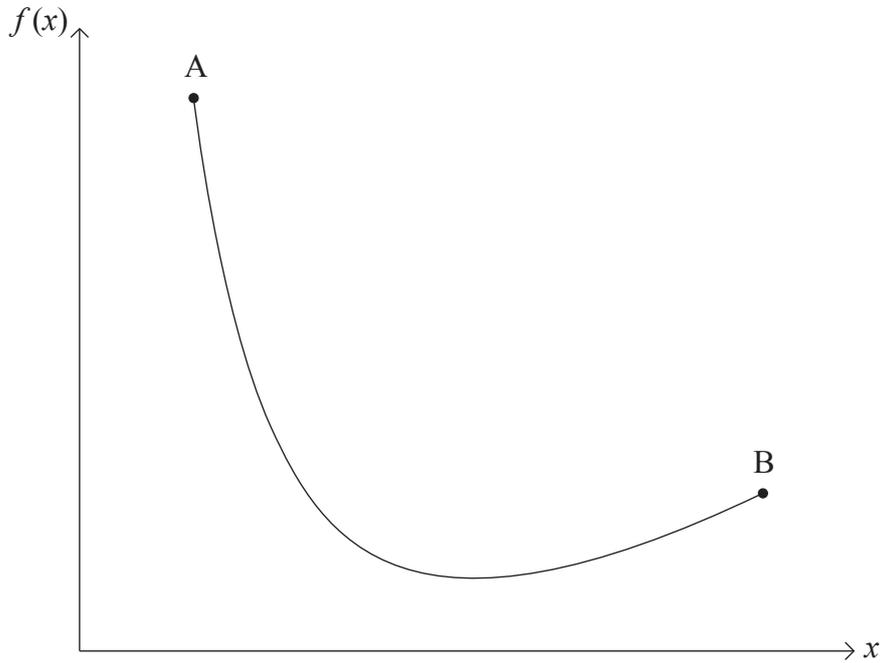
*(Question 3 continued)*

George decides to model the change in temperature of the coffee with a linear function using correlation and linear regression.

- (g) Use the **seven** results in the table to write down
- (i) the correlation coefficient;
  - (ii) the equation of the regression line  $y$  on  $t$ . *[4 marks]*
- (h) Use the equation of the regression line to estimate the temperature of the coffee at  $t = 3$ . *[2 marks]*
- (i) Find the percentage error in this estimate of the temperature of the coffee at  $t = 3$ . *[2 marks]*

4. [Maximum mark: 21]

The graph of the function  $f(x) = \frac{14}{x} + x - 6$ , for  $1 \leq x \leq 7$  is given below.



- (a) Calculate  $f(1)$ . [2 marks]
- (b) Find  $f'(x)$ . [3 marks]
- (c) Use your answer to part (b) to show that the  $x$ -coordinate of the local minimum point of the graph of  $f$  is 3.7 correct to 2 significant figures. [3 marks]
- (d) Find the range of  $f$ . [3 marks]

Points A and B lie on the graph of  $f$ . The  $x$ -coordinates of A and B are 1 and 7 respectively.

- (e) Write down the  $y$ -coordinate of B. [1 mark]
- (f) Find the gradient of the straight line passing through A and B. [2 marks]

M is the midpoint of the line segment AB.

- (g) Write down the coordinates of M. [2 marks]

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*(Question 4 continued)*

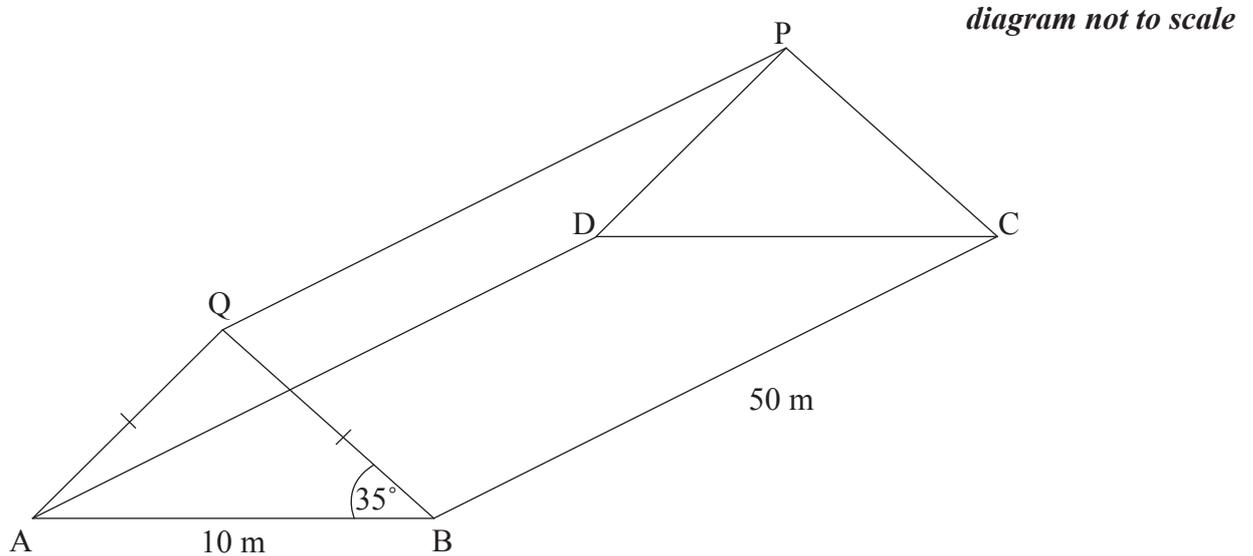
$L$  is the tangent to the graph of the function  $y = f(x)$ , at the point on the graph with the same  $x$ -coordinate as  $M$ .

(h) Find the gradient of  $L$ . *[2 marks]*

(i) Find the equation of  $L$ . Give your answer in the form  $y = mx + c$ . *[3 marks]*

5. [Maximum mark: 19]

A greenhouse ABCDPQ is constructed on a rectangular concrete base ABCD and is made of glass. Its shape is a right prism, with cross section, ABQ, an isosceles triangle. The length of BC is 50 m, the length of AB is 10 m and the size of angle QBA is  $35^\circ$ .



- (a) Write down the size of angle AQB. [1 mark]
- (b) Calculate the length of AQ. [3 marks]
- (c) Calculate the length of AC. [2 marks]
- (d) Show that the length of CQ is 50.37 m, correct to 4 significant figures. [2 marks]
- (e) Find the size of the angle AQC. [3 marks]
- (f) Calculate the total area of the glass needed to construct
  - (i) the two rectangular faces of the greenhouse;
  - (ii) the two triangular faces of the greenhouse. [5 marks]

The cost of one square metre of glass used to construct the greenhouse is 4.80 USD.

- (g) Calculate the cost of glass to make the greenhouse.  
Give your answer correct to the nearest 100 USD. [3 marks]